

# **Bridging between School Mathematics and Contemporary Mathematics: Turning a Dream into Reality**

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## **Abstract**

In many countries, school mathematics curriculum does not go beyond the 18<sup>th</sup> century mathematics. Any solution for bridging this gap must consider students' limited background, as well as teachers' time constraints. Our 'bridge' consists of periodically interweaving Mathematics-News Snapshots (MNSs), i.e., short descriptive presentations of recent mathematical results, throughout the teaching of the ordinary math curriculum during the three years of senior high school. More than 20 MNSs are already available (see <https://mns.org.il>). Our two-part workshop is aimed at sharing our solution. This will include a discussion of its underlying principles, a reverse engineering analysis of sample MNSs vis-à-vis the MNS authoring guidelines, an overview of three teacher preparation models, and results of our implementation follow-up studies. Finally, in the spirit of the conference, we'll invite attendees to adopt our solution, and possibly also to participate in developing more MNSs, thus turning our dream of bridging the gap into reality.

## **School mathematics Vs. contemporary mathematics**

Why do 21<sup>st</sup> century school mathematics curricula in many countries barely go beyond the 18<sup>th</sup> century mathematics? There are at least three reasons to this phenomenon – all seem to be related to the very nature of the discipline. Mathematics is a hierarchical discipline, it is an accumulative one, and it is amongst the most prolific in terms of new results obtained daily around the globe and added to the “common knowledge” through publication in approximately 550 professional journals (MathSciNet, 2019). Consequently, school math curricula lag behind, and the gap keeps growing wider.

Should this gap be of concern to mathematics educators? Those who believe, as we do, that high school graduates should be prepared for life in our modern era, must find it necessary to introduce *all* high school students to the abundance of mathematical accomplishments, or at least give them “a taste” of what mathematicians actually do. Much like viewing mathematics as a process and not just as results, high school graduates should be aware of the ways in which mathematics is constantly evolving, enriched and applied, rather than merely admire it, quite blindly. No matter what their future occupation may be, they should develop, and be equipped with, an image of mathematics as a vivid, rich, and intellectually satisfying area of human endeavour.

Narrowing the gap is a multifaceted challenge. In this presentation we share an attempt to meet it, thus turning the seemingly impossible into a possibility.

## **The challenges of narrowing the gap**

Any method for introducing *all* high school students to contemporary mathematics should first and foremost consider students' limited mathematical background. This gap is obviously a major obstacle limiting students' access to

an in-depth excursion into modern mathematics. School curricula are usually well-structured and most often aim at preparing students for school-leaving exams or college entrance qualification. Any method for introducing all high school students to contemporary mathematics must consider ways for interweaving news from contemporary mathematics in the required curriculum with no harm to students' progress in learning it.

Furthermore, mathematics teachers are usually under pressure to "cover" the curriculum in a limited amount of class time, not to mention the heavy load of professional life. As a result, most teachers cannot allocate time for following the current mathematics literature, let alone for adapting a news item for their students.

### **A solution**

Taking into account these limitations and constraints, we decided to provide high school mathematics teachers with ready-to-deliver Mathematics-News Snapshots (MNSs). The teachers are expected to interweave these MNSs in the school curriculum on a regular basis throughout Grades 10-12. Each MNS is a PowerPoint presentation of a mathematical result published in the professional literature in the past few decades. It is developed by teams of professional mathematicians and mathematics educators according to specific guidelines. None of the MNSs is a complete and in-depth treatment of the particular news. They are designed to provide a 'glass-bottom boat tour' over the deep, colourful, vivid, and overwhelming ocean of mathematics, without 'getting wet'.

More than 20 MNSs are already available (see <https://MNS.co.il>) and more are being developed. The MNSs are revised and updated annually in light of feedback from teachers and new relevant publications.

### **Math News Snapshots preparation guidelines**

Movshovitz-Hadar (2008) suggested a classification of mathematical news items into five categories:

- i. A recent (interesting) problem and possibly its solution.  
E.g. Herzberg and Murty's (2007) paper concerning the mathematical problems related to Sudoku puzzles, and Murty's (2008) discovery of a Sudoku puzzle with exactly two solutions.
- ii. Long-term open problems recently solved.  
"Recently solved" was defined as the last 30-40 years and "long-term" as at least 100 years.  
E.g. The proof of Kepler's conjecture; The mapping of the E8 Lie group; The Four-Color Problem; Fermat's Last Theorem.
- iii. A recently revisited problem.  
This category includes a new proof to a known theorem, new findings in an already solved problem, a new solution to a previously solved problem, a generalization of a well-established fact, or even a salvaged error.

- E.g. In 1996, four mathematicians published a proof of the Four-Color problem, clearing the doubts about Appel and Haken's computer-assisted proof, provided twenty years earlier (Robertson, Sanders, Seymour, & Thomas, 1996). Another example is the current findings of the constant race for new prime numbers via GIMPS: Great Internet Mersenne Prime Search.
- iv. A mathematical concept recently introduced or broadened, including concepts that evolved into new areas in mathematics.  
E.g. The concept of dimension, broadened in 1918 by Felix Hausdorff and later on by Mandelbrot (1977) to non-integer dimension and fractals; The notion of Vertex Algebra invented by Richard Borcherds upon which he based his proof for the Conway-Norton "Moonshine conjecture", for which he received the 1998 Fields Medal (Borcherds, 1992; Singh, 1998).
- v. A new application to an already known result in mathematics.  
Mathematics developments are driven mainly by human curiosity, quite often independent of the physical real world. It is fascinating to find out that a piece of pure mathematics, developed by some intellectually-intrigued mathematicians, becomes utterly useful for some real application. Perhaps the ultimate example in this category is the employment of prime factorization to modern Public-Key-Cryptography (Rivest, Shamir, & Adleman, 1978). Another nice example is the Linear Algebra application to Google search engine (Berman & Shaked-Monderer, 2009), and a recent one has to do with advances in optimization theory bringing Hilbert's work into a world of self-driving cars (Hartnett 2018).

### **MNSs Design-Principles (DPs)**

The MNS design principles involve three categories (Amit and Movshovitz-Hadar 2011):

#### 1. Mathematical content

Each MNS should include:

- a. A recent mathematical result (of one of the 5 categories mentioned above) published in the professional mathematics journals in the past few decades, and possibly also yet unsolved related problem(s) related to the new result;
- b. The relevant school mathematics to make the result accessible to high school students, given their limited knowledge;
- c. Some relevant advanced and/or extra-curricular mathematics, possibly including a few new concepts and terminology.

#### 2. Meta-mathematical content

Each MNS should include:

- a. Elements reflecting the nature of mathematics, such as the nature of definition, proof, and truth; the nature of problem posing and problem solving; search for patterns; generalization;
- b. Historical background relevant to the news such as long-term conjectures, failures and success in proving them, and cultural background if relevant;

- c. Details about mathematicians: Either about their unique contribution or biographical details about such mathematicians, such as their relationship with the professional community and with the public;
- d. Connections to some area within mathematics or to some topic in the curriculum; Connections between mathematics and other disciplines or to everyday life or to any other students' common interests;

### 3. Pedagogy.

- a. Each MNS should take the form of a PowerPoint presentation consisting of a narrative and visual aids (Photos, diagrams and animated illustrations) that describe, demonstrate, illustrate or represent mathematical ideas, concepts and other elements;
- b. The narrative should take the form of expository statements enabling the teacher an accurate and concise presentation in a Socratic dialog (Q & A) mode.
- c. The language should be simple and friendly, concrete and non-formal as much as possible, bridging between students' level of mathematics and the advanced level of the mathematical news presented.
- d. Duration should be limited to about 25 minutes so as not to take up too much of the class instruction time;
- e. The MNS will include a list of suggestion for further reading.

The development of each MNS takes the following steps: (1) Choosing an appropriate result from the literature; (2) searching for papers about the selected news, the target audience of which is amateurs or non-professional mathematicians; (3) Developing the outlines of "the story", in the form of a sequence of questions and answers, with emphasis on building up suspense and surprise; (4) Searching for related photos, illustrations and applets, as well as bibliographical details about the mathematicians involved; (5) Preparing the PowerPoint presentation on a template, keeping in mind the guidelines mentioned above. This step involves an exchange of several drafts between the authors; (6) Field testing and revising the PowerPoint presentation before handing it out to teachers.

### **A reverse engineering analysis of sample MNSs vis-à-vis the MNS design principles**

To familiarize the user with the unique features of the MNSs we developed a tool named "reverse engineering". Participants in our Ireland conference workshop will employ this tool subsequent to navigating in the MNS website. Each participant will browse through the slides of one MNS of their choosing and identify in each slide the relevant design-principle/s, using a reverse engineering worksheet we will provide.

### **Teacher preparation models and Implementation follow-up studies**

The MNSs project was launched in 2008 as an action research carried out by a single teacher interweaving an early version of nine MNSs in four classes of various levels of ability (Amit, Movshovitz-Hadar & Berman, 2011). The second phase involved a weekly clinical work with the whole team of mathematics

teachers in three high schools (supported by the foundation of the Israel Academy of Science). The third phase involved an open call for participation in an on-line professional development program for high school mathematics teachers which took place in after school hours during the school year. Follow-up studies in all these preparation models were focused mainly on the teachers. To establish the impact on the students, we started in school year 2016/17 a longitudinal study of the effect of interweaving MNSs for 3 consecutive years of senior high school. The call for teachers to take part in this study attracted 47 teachers who had not participated in any preparation prior to this one, and 14 continuing teachers who took part in previous phases. This study is currently ongoing (supported by the Ministry of Science). The responses of both teachers and their students are carefully documented, and we hope to establish the impact of this intervention on students' perceptions of mathematics in a report due in April 2020. Following this move we plan to call the Israeli Ministry of Education to adopt the MNSs as a part of the Israel mandatory school mathematics curriculum.

### **An invitation to join**

In the spirit of the Ireland 2019 conference, we invite attendees to adopt our solution, and possibly also to participate in developing more MNSs, thus turning the bridging-the-gap dream into reality.

### **Acknowledgement**

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