

Is it our responsibility to introduce high school students to contemporary mathematics?

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Abstract

Should high school mathematics teachers give their students a taste of contemporary mathematics? In this article, obstacles and a suggested solution are provided to enable teachers to incorporate contemporary mathematics into their programs.

Introduction

“...One of the big misapprehensions about Mathematics that we perpetuate in our Classrooms is that the teacher always seems to know the answer to any problem that is discussed. This gives students the idea that there is a book somewhere, with all the right answers to all of the interesting questions, and that teachers know those answers. And if one could get hold of the book, one would have everything settled. That's so unlike the true nature of mathematics...” (Henkin, 1981)

What is the true nature of mathematics?

What follows are five illustrative examples.

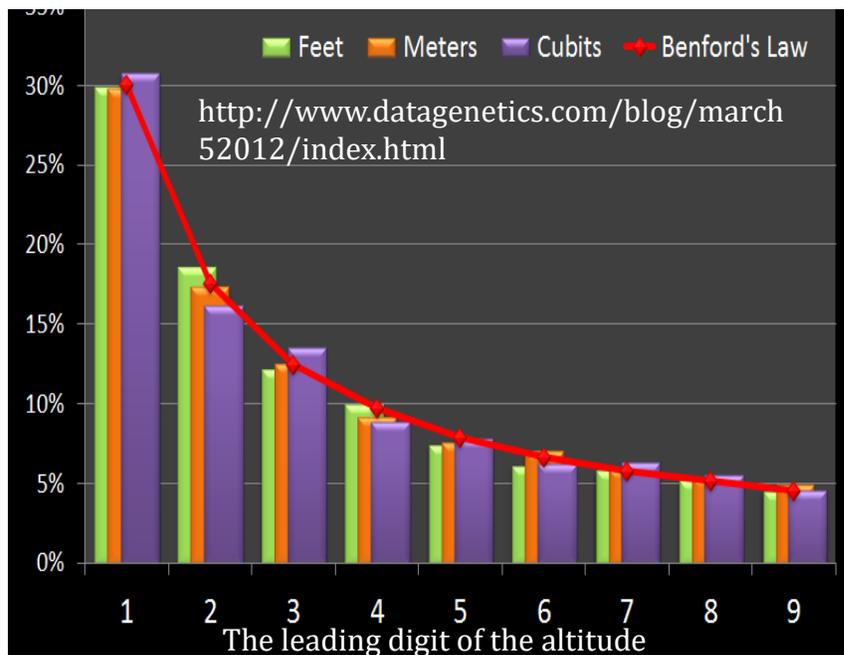
1. The centuries-old Twin Primes Conjecture (TPC) posits that there are infinitely many consecutive primes, i.e., pairs of primes separated by exactly 2 (e.g., 11,13; 17,19). This conjecture seems to stand contrary to our intuition, rooted in the general tendency of gaps between adjacent primes to become greater as the numbers themselves get larger. On May 13, 2013, a lecturer at the University of New Hampshire showed that you would never stop finding pairs of primes separated by 70 million (or less). He was the first to establish the existence of a finite bound for prime gaps. This finding gave a start to a mathematical race. On Nov 19, 2013, a researcher at the University of Montreal, pushed the gap down to 600 (Meynard, 2013). On April 14, 2014, the gap was reduced to 246 and later to 12, then to 6. How soon will the gap be reduced to 2, thus proving the TPC? – This remains to be seen. (For more details, see Wikipedia Twin Prime.)
2. In 1611 Johannes Kepler stated that even the densest packing of spheres of equal diameter takes no more than ~74% of the space. For more than 400 years, the mathematics community struggled to prove it, in vain. There seemed to be a breakthrough when, on August 10, 2014, a team led by a mathematician at the University of Pittsburgh, announced that their decade-long effort to construct a computer-verified formal proof of the Kepler conjecture, was complete. This 400-year-old best sphere-packing conjecture had finally become a theorem. Furthermore, the long struggle for getting this proof recognized as valid beyond doubts paved the way for a new era of mathematics, in which machines will do the “dirty work” of verifying proofs before their publication in a respectful journal and leave mathematicians free for deeper thinking. (Hales et al., 2017)

3. Take a look at the first 40 elements of the well-known Fibonacci series: $a_1 = a_2 = 1$ and for $n > 2$ $a_n = a_{n-1} + a_{n-2}$: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 11946, 18711, 30657, 49368, 80025, 129393, 209418, 338811, 548229, 887040, 1435269, 2322309, 3757578, 6079887, 9837465, 15917352, 25754817, 41672169, 67426986, 109099155...

Now, let's examine the occurrence frequencies of the leading digits (the left-most digit) in the part of the sequence shown above. Would you expect a difference?

Leading digit	1	2	3	4	5	6	7	8	9
Frequency	11	7	6	3	3	4	0	4	2

Are you surprised? This unique phenomenon of the uneven distribution of the leading figure is known as Benford's Law. One finds it in many other series such as $a_n = n!$ and $a_n = 2^n$. And surprisingly, it is true of much statistical data, as well. Look at the distribution of the leading digit of the altitudes of the top 122,000 populated cities shown in the graph below.



The Benford law was applied to forensic sciences. In 1993, *Wayne James Nelson* was found guilty of trying to defraud *The State of Arizona* of nearly \$2 million by diverting funds to a bogus vendor (*State of Arizona v. Wayne James Nelson CV92-18841*.) He selected payments carefully, intending to make them appear random: None of the check-amounts were duplicated, there were no rounded numbers, and all the values included numbers of dollars and cents. The poor guy failed to realize that his seemingly random looking selections, were far from behaving as expected by Benford's law! (Durtschi et al., 2004.)

4. In 1975, in his well-known “Mathematical Games” column in *Scientific American*, Martin Gardner surveyed all that was known on tessellating the plane with convex polygon tiles. Among others, he referred to regular pentagons that do not tessellate the plane (why?), and to the study of non-regular pentagons that do tessellate. At that time, eight families of pentagons were known to facilitate the tessellation of the plane, and a prominent mathematician claimed that there were no more. However, soon after his paper was published, one of the readers, a computer programmer, discovered the 9th type of pentagons that tessellate the plane, which falsified the mathematician’s claim and reopened the search for more types (assuming there are more).
Indeed, two years later, a housewife from California, blending her love for mathematics and her love for art, discovered four additional types of pentagons that tile the plane (Schattschneider, 1978). In 1985, a German student found an additional type, raising the number of types to 14. No others were found for 30 years. Then, in October 2015, the following title appeared in *The Guardian*: "Attack on the pentagon results in discovery of new mathematical tile." (<https://www.theguardian.com/science/alexs-adventures-in-numberland/2015/aug/10/attack-on-the-pentagon-results-in-discovery-of-new-mathematical-tile>). The article described the discovery of a 15th type of tiling by a group of young mathematicians from the University of Washington. Yet the general question remained open: Are there additional types of pentagons that can be used to tile the plane? How many? In the summer of 2017, a 37-year-old French mathematician, published a computer proof that there *are no more* than 15 types of convex pentagonal prototiles, thus bringing the drama to closure (Rao, 2017). (See all 15 types here: <http://www.wolframalpha.com/input/?i=pentagon+tiling>).
5. Perhaps the best illustration of the creative nature of mathematics is numerical. According to MathSciNet, AMS database of reviews, abstracts, and bibliographic information for much of the mathematical sciences literature, “over 125,000 new items are added each year, ... [and] over 90,000 reviews are added to the database each year... Reference lists are collected and matched internally from approximately 650 journals.”

Back to Henkin’s quote that opened this article

In most cases, students graduate high school without any exposure to contemporary mathematics in the making. They go out to their next stage in life, having the (wrong) impression of mathematics as a “dead-end” boring discipline in which all answers are known (to the teacher), and nothing is left open for them to stimulate their intellectual curiosity or creativity. This impression is largely due to the very nature of mathematics as an accumulative and hierarchical discipline, so that one cannot be expected to understand accurately and deeply the details of contemporary mathematics without learning first the past achievements which never become obsolete. Hence, mathematics curricula in many countries do not reach beyond the 18th-century mathematics, and contemporary mathematics belongs to tertiary education. Covering the mandatory curriculum, preparing students for school graduation exams or university admission requirements, is a stressful job that leaves very little freedom for the teachers to toy with the idea of exposing their students to more than required.

Why should we care?

Is it worth our time to invest in attempting to cope with this seemingly impossible task? After all, there are so many other issues for the mathematics education community to worry about. Well, there are several good reasons to care about the lack of exposure to modern mathematics in school. As a result of it, only a small percentage of high school graduates consider tertiary education in mathematics or in mathematics-related professions, although the job market in these areas is very promising. This, in turn, has an impact on the economy. Moreover, it perpetuates the ignorance of the adult society about mathematics. Rather than being literate about mathematics and its impact on the quality of our lives in the 21st century, most adults could not care less about mathematics once they leave high school. At best, they admire blindly those “weird” ones who chose to become mathematicians, having no idea what it means to do mathematics.

The gap between school mathematics and contemporary mathematics, is it a necessary evil?

Can the high school mathematics teacher be expected to be able to give high school students a taste of contemporary mathematics without harming the progress in teaching the heavily loaded mandatory curriculum? And facing the increased effort it takes to keep up-to-date in contemporary mathematics?

How might it be possible to communicate the true nature of contemporary mathematics to ALL high-school students without harming their progress in learning the heavily-loaded mandatory curriculum? And facing students’ insufficient backgrounds required to delve into the depth of modern mathematics?

A few questions to ponder before we attempt at a solution

How do we make young children curious about written words? -- By reading a story to them and pointing at the printed words; By teaching them how to read and write; By exposing them to classic books. We do not do it only because reading and writing are essential for functioning in the daily life of the adult world. Nor do we do it because we expect all of them to become poets or book authors. We do want to open for them the door to reading the literature, because it is a great part of our culture, deeply rooted in generations of human creativity.

How do we make children like music? -- By turning on some machine that plays some music, we believe would be pleasant to their ears; By teaching them how to sing a song; By suggesting that they learn to play an instrument. We do not do it because they need it for any practical purpose. We do not expect all of them to become professional musicians. We do want to open a window to music for them because it is a rich part of our culture.

How do we make students appreciate the performing arts? -- By taking them to the theater to watch a play, we believe it would touch their soul; By encouraging them to put on a costume and perform some role and enjoy it. We do not do it because we aim at training them as actors or dancers. We do it because we want to expose them to this wonderful part of human culture, of human creation over generations.

How can we make our students curious about mathematics, like mathematics, enjoy it, and appreciate its impact without expecting many of them to become mathematicians? Have we gone a bit too far with the emphasis on learning-by-doing and problem-solving? Sometimes a piece of mathematics you read or hear about can be so neat and elegant that you feel the AHA effect, and it makes you want to shout, "Got it!" (even if you did not discover it yourself!) Do you enjoy hearing a clever solution to an interesting mathematics problem you were not able to figure out on your own? Do you find a well-written expository paper interesting and fulfilling? Do you like a vivid presentation of a counterintuitive mathematical result that is not familiar? Why deprive school students of these pleasures?

The Math News Snapshots Project

The Math News Snapshots (MNS) Project is built on the premise that high school students should not be deprived of these pleasures. They should have the opportunity to learn about contemporary research in mathematics as part of their education, because it is a part of human culture. Quite often, the hierarchical nature of mathematics does not make contemporary mathematics accessible to students' active learning. On the other hand, ignoring the vivid and prolific nature of contemporary mathematics leaves students thinking that all mathematics has been "done" already. Consequently, students often do not understand the work of living mathematicians. We argue that learning about new findings in mathematics and new applications of mathematics is as necessary for widening students' mathematical horizons as having them experience the joy of problem-solving, firsthand.

MNS lessons provide a glimpse into contemporary mathematics tailored to be accessible to high school students. Each MNS is in the form of a PowerPoint presentation. It is focused on a single result, along with its background, implications, and applications where relevant. (Please see the project website at <https://mns.org.il/> for examples.) Interweaving MNSs one at a time throughout the teaching of the high school curriculum is worth trying. It can have a significant effect on the way students perceive the true nature of mathematics.

Note: The MNS project started in Israel in 2012 and has become well established there. In 2017-2019, a longitudinal study supported by the Israeli Ministry of Science was conducted to investigate the long-term effects of introducing high school students to mathematical news on a regular basis throughout their three years of high school (Movshovitz-Hadar et al., 2019). Currently, our MNS team is looking for pilot sites and partners who are interested in bringing MNSs to U.S. classrooms. Please contact Nitsa Movshovitz-Hadar at nitsa@technion.ac.il to learn more about the project.

Resources and recommendations for further reading

Math New Snapshots Website: <https://mns.org.il/>

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Nitsa Movshovitz-Hadar, Ph.D., is Professor Emerita at the Technion – Israel Institute of Technology. She began her career as a high school teacher, and after 12 years, decided to devote her time to teacher education, curriculum development, and related research. In 1975, she received her Ph.D. at the University of California - Berkeley. After completing this program, she returned to the Technion to establish the Research and Development Center for Mathematics Education. She has also served as Director of MadaTech, the Israel National Museum of Science, and is a member of the advisory committee for MOMATH. Nitsa has led several research projects, including the most recent, Math News Snapshots project. To date, this project has developed 24 PowerPoint presentations, each focusing on one new result in mathematics, and making those accessible to high school students via their classroom teachers.